To Study AC Nuclear Stark Effect Via Super-Intense laser-ion Interaction

Master of Philosophy
In
Physics

By

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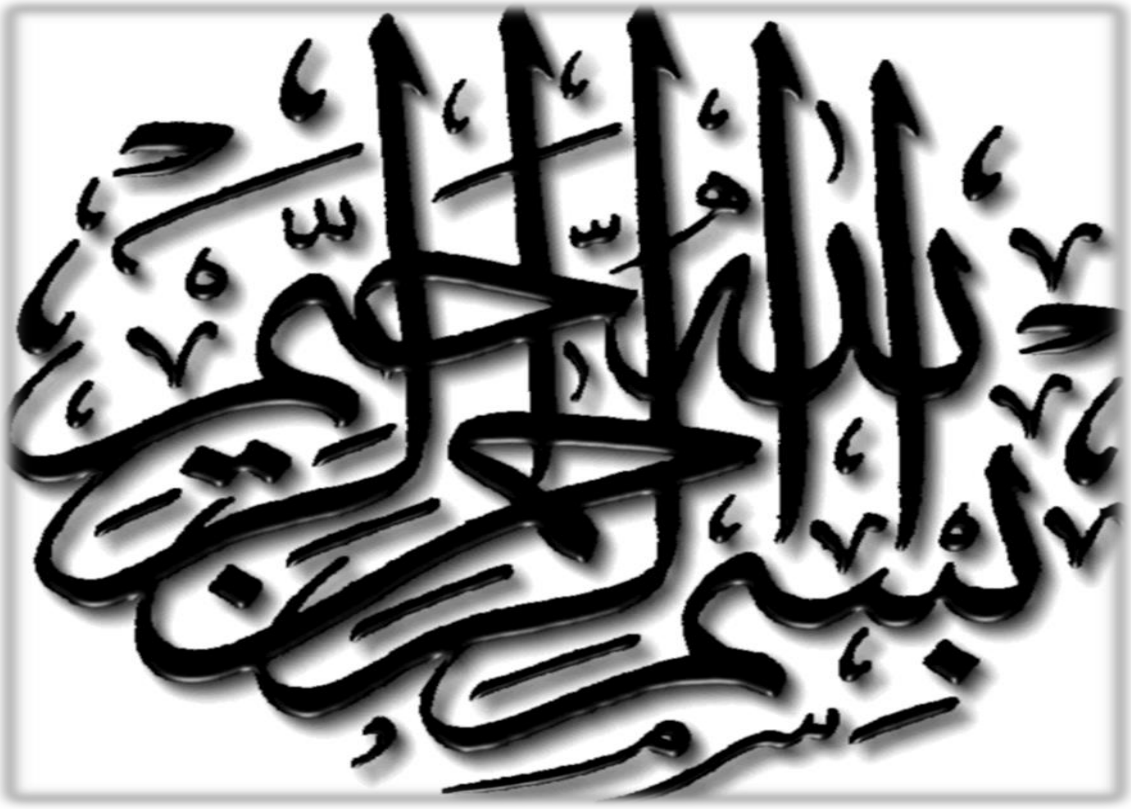
بسم الله الرحمن الرحيم

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Declaration

I Mr. **Ali Raza Mirza**, Roll No. **0419-MPHIL-PHY-13**, student of **M.Phil.** in the subject of **Physics**, session **2013-2015**, hereby declare that the matter printed in thesis titled, **To study AC nuclear Stark effect via super-intense laser-ion interaction** is my own work and has not been printed, published and submitted as research work, thesis or publication in any form in any University, Research institution etc. in Pakistan or abroad.

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RESEARCH COMPLETION CERTIFICATE

Certified that the research work contained in this project dissertation titled **To study AC nuclear Stark effect via super-intense laser-ion interaction** has been carried out and completed by **Ali Raza Mirza Roll No. 0419-MPHIL-PHY-13** during his **M.Phil.** in the subject of **Physics** under my supervision.

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Dedicated to my beloved

Mother

who sent me on the path towards intellectual pursuit and whose prayers and Efforts are great source of strength to me in every noble venture.

ACKNOWLEDGEMENTS

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Thank you all…

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ABSTRACT

Since the available laser is not such to probe the bare nucleus because the wavelength of available laser is much larger than the size of nucleus. As we shine laser on hydrogen atom or hydrogen-like ions, electron starts oscillating near the nucleus. Due to the oscillations of electron the nucleus experiences periodic electric field which is actually the source of AC Nuclear Stark effect. We will calculate analytically the Stark shift in the energy levels of Hydrogen atom nucleus under the action of super-intense laser.

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Chapter 1

Introduction

ALLAH gifted moral sense to human being which is the ever best thing that we are blessed. Other than human being all the other creature are deprived of such beautiful property. The world in which we are living is full of ambiguous and when man observes these things near it, his curiosity urges him to explore the nature. So he started to think over the nature from his birth and his research takes him to the discovery of so many things. And these inventions have improved our living standard very much. This is what the greatest ALLAH wants us to do that explore the nature and work for the benefit of human being. Research work in science is also an attempt to serve the humanity in benefitting manner.

To meet the energy crises over the whole world one must need to study the matter very deeply. There are so many scientists in the world which are working on different substances to produce energy from them. Study of atom mean the study of its properties and this work also a kind of study of the properties of H-atom. Hydrogen atom is one of the simplest element among all the elements which have been discovered yet in the nature. The branch of physics which deals with the study of atom and its constituents is Atomic and Nuclear physics in which we use Quantum Mechanics as a basic tool. Using Quantum Mechanics we can discuss the motion of tiny particles like electron, proton and neutron and also their interactions with the energy. One can think of light-atom interaction, when an atom is placed in external energy source say light, its energy will definitely be changed. According to Bohr model, for any bound state of an atom there exists definite energy levels and these levels will be shifted by little amount when the system is placed in external electric or magnetic field. For example if we place H-atom in a laser field, the energy shift in atomic energy level is observed this effect is known as Stark effect. As ground state nucleus of an atom is also a bound state so it also possesses definite energy levels. This idea is based on experimental facts and on different nuclear models. So we will see that there is also an

energy shift in the levels of nucleus with the help of laser. This phenomenon is termed as Nuclear Stark effect by an intense laser field.

When we shine intense laser on hydrogen atom, its electron starts oscillating near the nucleus in the electric field of laser with a particular frequency ω_o. The electron oscillations can completely be controlled by the laser parameters that is frequency ω and the strength E.

When an electron oscillates under laser, nucleus feels the periodic electric field provided by the electron due to its oscillating motion. As nucleus experiences such sinusoidal field its energy levels split up.

In this thesis Chapter 2 consists of discussion on all possible effects when a laser interacts with the atom. In laser atom interaction, multiple phenomena other than Stark effect are completely described in this chapter.

In chapter 3 linear and quadratic Stark effects at atomic level are discussed, also we develop a complete mathematical formulism for AC Stark effect which may help us to study the Nuclear Stark effect.

In Chapter 4 we discuss about nucleus especially its energy levels. There are different nuclear models which confirms that nucleus has specific energy levels. This idea is also supported by the experimental facts.

In Chapter 5 we thoroughly develop a formula which will be helpful for us to calculate of energy shift in H-atom nucleus when it interacts with the laser of different strength and different frequency.

Chapter 2

Laser Atom Interaction

Laser is a electromagnetic radiation which is very important thing to investigate the dynamics of tiny particles. When we fall laser upon an atom, its energy makes significant change in the properties of that atom such that shift in the atomic levels, various phenomena may occur which are given below in succession.

2.1 Atomic Excitation

By excitation we mean that when we supply a specific amount of energy to a system, as a result electron in atom captivates that quantity of energy and jumps from lower energy level to higher energy level, we can say that atom has got excited. In laser atom interaction, we add energy to the system by laser source. Laser photons are monochromatic and coherent, when they are incident on an atom then its electron jumps to one of the higher energy states by absorbing laser photons, this process is called atomic excitation. The particular extent of energy required to excite an electron is known as excitation energy.

As we see there exists finite probability of excitation of hydrogen atom when we shine intense attosecond laser on it [1]. Figure 2.1 shows atomic excitation, electron leaves its ground state by absorbing multiple photons from laser to excite the atom.

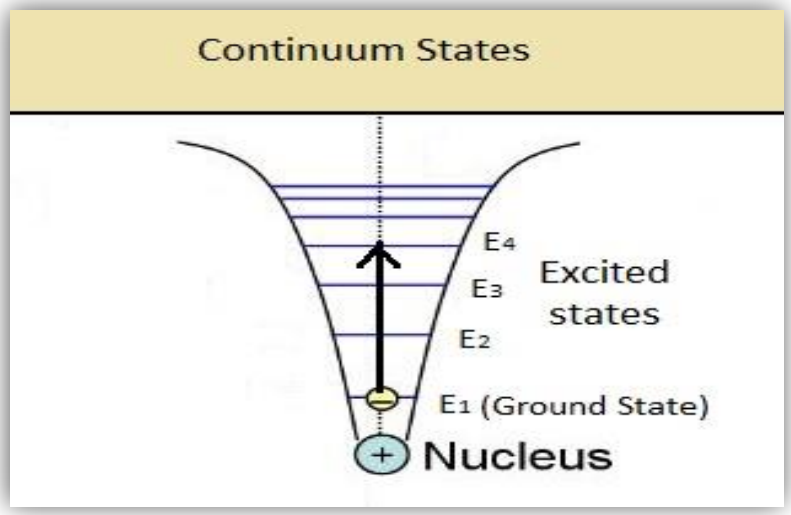

Figure 2.1: Excitation of an atom by laser

There are two conditions for atomic excitation with laser;

1. Energy of the photons taking part in excitation must be less than the ionization energy of the system.
2. $\Delta E = n\hbar\omega$, where "$n\hbar\omega$" is the energy of the laser carrier photons. And "ΔE" is the energy difference between ground state and one of the excited states of the system.

Below than ionization energy, if $\Delta E \neq n\hbar\omega$ the electron will not captivate laser energy and atom will not be get excited. Once an atom is excited, it de-excites to its initial state along with the emission of photons.

2.2 Multiphoton Ionization (MPI)

The process in which an atom gets charged either by gaining or losing electrons to make respective ions is termed as ionization. To add or remove electron from atom or molecule, in both cases we need specific amount of energy. The minimum amount of energy needed to add or remove an electron from its outermost shell is known as ionization energy.

Multiphoton ionization will occur if $n\hbar\omega > I_p$, where $n\hbar\omega$ is the net energy absorbed by electron and I_p is the ionization energy.

Initially it was believed that the photon of energy less than the ionization energy of targeted atom cannot release the electron from its bound state. Few years later after the invention of laser, it was observed that the phenomenon such as ionization can be occurred in the presence of large number of coherent and monochromatic photons. No doubt it contradicts the Einstein explanation of photoelectric effect but at that time there was no concept of laser.

How many photons the system will absorbs from incident laser? It totally depends upon intensity of the laser field, more the intensity more will be the absorption and hence greater the probability of ionization. In other words we can say that probability to capture number of photons totally depends upon the number of photons per unit time incident on the system. Upon the absorption of greater number of photons causes to decrease the ionizing time of the system. As the system absorbs photon its life time is given by the Heisenberg uncertainty principle.

$$\Delta E.\Delta t \geq \hbar \tag{2.1}$$

Where ΔE is the uncertainty in energy after the absorption of photons, actually in between the initial and final states, there are infinite number of virtual states which exist for very short time Δt within oscillating field of laser.

By successive absorption of photons, going through virtual states electron leaves its bound state hence system gets ionized. Schematic diagram of multiphoton ionization is shown in figure 2.2.

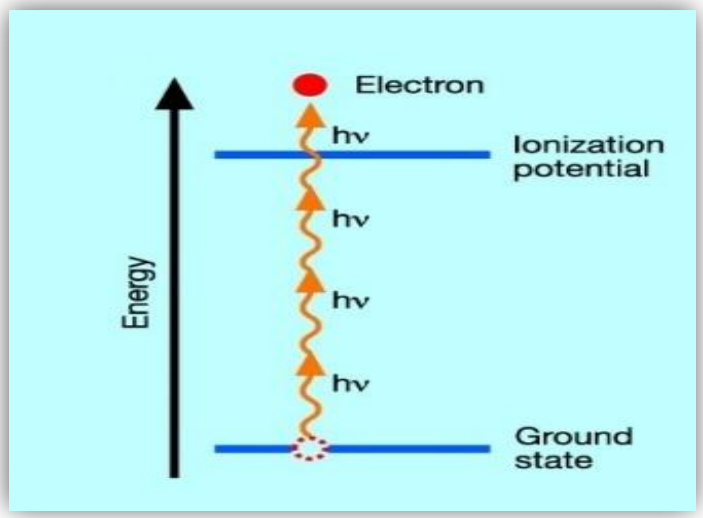

Figure 2.2: Multiphoton Ionization

We can calculate the kinetic energy of the electron separates the system by following formula.

$$K.E = n\hbar\omega - I_p \tag{2.2}$$

where I_p is the ionization energy of the system and n is the number of absorbed photons.

Multiphoton ionization in Xe has been observed firstly in 1987 with 100-fs laser pulses of intensity 1.7×10^{14} Wcm^{-2} by the absorption of seven photons [2].

We observe multiphoton ionization in metals mostly, where electron absorbs multiple photons to make the system ionized. If I is the intensity of laser and σ_n is the cross sectional area with the absorption of n number of photons, then ionization rate can be calculated by following formula:

$$\Gamma_n = I.\sigma_n \tag{2.3}$$

2.3 Above Threshold Ionization (ATI)

A very similar phenomenon like multiphoton ionization is the Above Threshold Ionization in which electron absorbs few more photons than the number of photons essential for MPI. The total energy absorbed by an electron from laser can be written as

$$E = (n + k)\hbar\omega \tag{2.4}$$

Where n is the number of photons required for ionization and k is the additional number of photons which are the source of kinetic energy of the ionized electron. Here the energy of single photon is less than energy required for ionization so it is again a multiphoton process. Diagram 2.3 shows such process.

ATI process has been observed in Xe in 1987 with laser pulses having wavelength 616 nm [3].

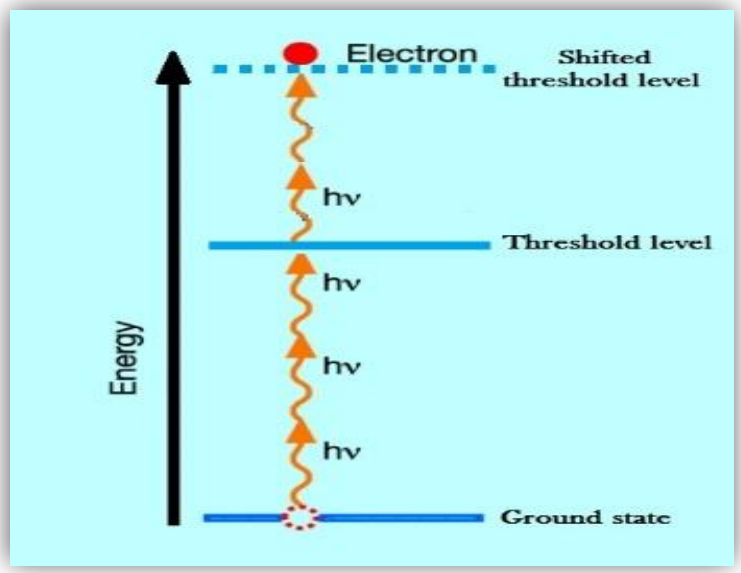

Figure 2.3: Above Threshold Ionization

We can increase or decrease the energy of electron by laser parameters as shown by following formula.

$$E = \frac{2e^2\mathcal{E}^2(t)}{m\omega^2} \tag{2.5}$$

So electron will absorb greater number of photons by increasing the intensity and decreasing the frequency of incident laser field.

2.4 Tunneling Ionization (TI)

Tunneling Ionization is very important quantum mechanical phenomenon which can be observed in laser atom interaction. In such situation electron does not gain sufficient energy to compete the Coulomb barrier. Under the action of sinusoidal electric field of laser, the wings of Coulomb potential are suppressed so there is finite probability of electron to tunnel through the potential barrier as shown in figure 2.4.

The phenomenon of high harmonic generation (HHG) is carried out through tunneling ionization. As potential barrier is distorted with the help of laser so width decreases and height of the barrier increases which increases the tunneling probability.

The tunneling probability is decided by the quantity known as Keldysh parameter [4], which is given by

$$\gamma = \frac{\tau}{t} \tag{2.6}$$

where τ is the time required for tunneling and t is the laser time. For γ<1, which we mean tunneling time is less than laser time *i.e.* electron gets enough time to tunnel in the presence of laser, hence tunneling dominates. For γ>1, ATI dominates where laser does not provide enough time to tunnel out. Keldysh parameter can also be defined as

$$\gamma = \left(\frac{I_p}{2U_p}\right)^{1/2} \tag{2.7}$$

where I_p is ionization potential and $U_p = \frac{e^2\varepsilon^2(t)}{4m\omega^2}$ the ponderomotive energy of electron, which is defined as the average energy gained by electron in laser field. Keldysh showed that TI and MPI are the limiting cases of ionization [5]. Tunneling ionization in noble gasses has been studied in 1989 with the help of 1-μm, 1-ps laser pulses of intensity 10^{16} W/cm^2 [6].

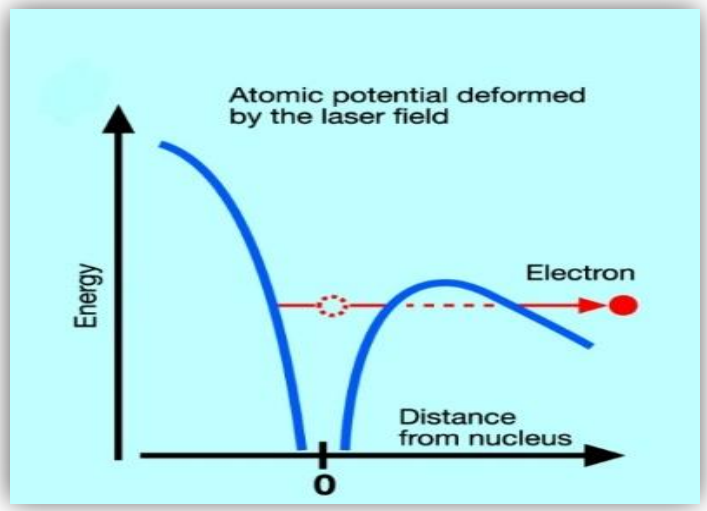

Figure 2.4: Tunneling Ionization

2.5 Over the Barrier Ionization (OBI)

As we have learnt already the Coulomb potential barrier can be distorted by the laser, so we may sufficiently increase the laser intensity so that height of the barrier faced by the electron becomes zero and then electron can easily escape walking over the barrier, this process is termed as over the barrier ionization. Schematic diagram for OBI is shown in figure 2.5. In such process all of the laser energy is utilized to distort the potential barrier so electron feels free to leave the system.

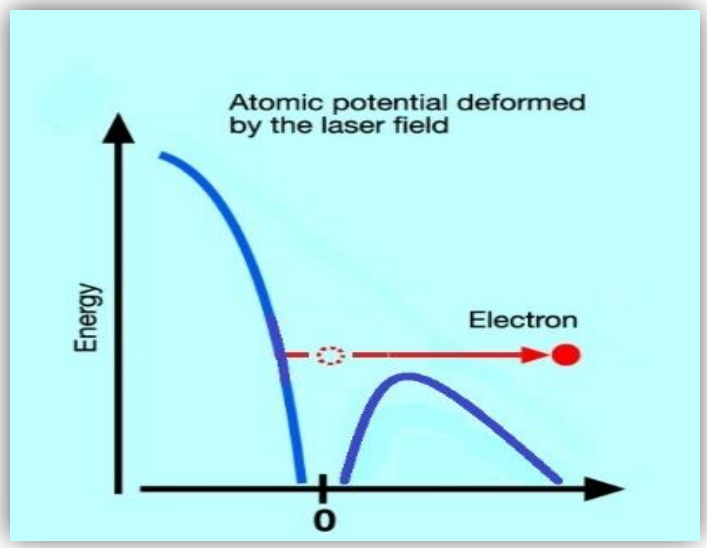

Figure 2.5: Over the barrier Ionization

2.6 High Harmonic Generation (HHG)

In HHG, photons of high energy are produced when an atom is exposed to laser beam, the harmonics obtained are the integral number of original harmonics. This process is similar to TI but here electron is bound to recombine with its parent nucleus after tunneling ionization. The idea of HHG was given by Franken *et al.* in 1961 for the first time with the help of ruby laser [7]. For this process laser of intensity 10^{14} W/cm^2 or more is required [8]. The harmonics of higher order had been observed by the interaction of CO_2 laser with plasma obtained from solid [9]. Now a days HHG has wide range of applications such as it can be used to generate the train of

attosecond pulses [10]. HHG process can easily be explained by Three-Step Model which is given below.

Ionization

The first step is ionization in which electron leaves its bound state through tunneling under the action of laser beam.

Acceleration

Electron reaches in continuum state where it accelerates freely and gains energy from incident laser field.

Recombination

When laser changes its cycle, electron is attracted by nucleus as well as pushed by laser field, as a result high energy photon is released upon recombination of electron with its parent ion.
Three-step model is shown in figure 2.6

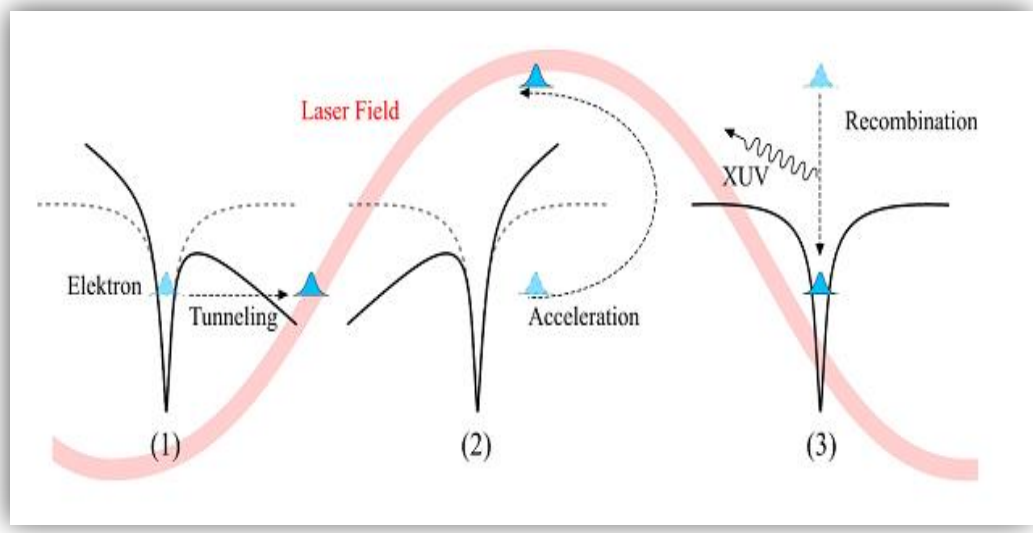

Figure 2.6: Three-step model

Chapter 3

Atomic Stark Effect

We should have idea about perturbation theory when we are going to study the interactions. Perturbation theory is a mathematical formulism that provide us approximate solutions when a system is subjected to perturbation. There are two kinds of perturbation theory described below in detail:

3.1 Non-degenerate Perturbation Theory

3.1.1 General Formulism

Upon solving the time-independent Schrodinger equation for 1-D infinite square well potential, we get a set of eigenfunctions Ψ_n^O (which are orthonormal to each other) corresponding to eigenvalues E_n^O.

$$H^O\Psi_n^O = E_n^O\Psi_n^O \tag{3.1}$$

Where

$$\langle\Psi_n^O|\Psi_m^O\rangle = \delta_{mn} \tag{3.2}$$

Subscript 0 showing the unperturbed quantities. Now let us perturb the system by a small amount H' and try to find the new eigenfunctions and corresponding eigenvalues

$$H\Psi_n = E_n\Psi_n \tag{3.3}$$

Now it becomes difficult to solve the Schrodinger equation for new complicated potential. So we will use **Perturbation theory** to approximate the solutions knowing the exact solutions for unperturbed system

The new Hamiltonian is

$$H = H^O + \lambda H' \tag{3.4}$$

Now we expand $E_n$and Ψ_n in power series

$$\Psi_n = \Psi_n^O + \lambda\Psi_n^1 + \lambda^2\Psi_n^2 + \cdots \tag{3.5}$$

$$E_n = E_n^O + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots \tag{3.6}$$

Where λ is very small number, it can have value between 0 and 1. Using these values in Eq. (3.3)

$$\left(H^O + \lambda H^{'}\right)[\Psi_n^O + \lambda\Psi_n^1 + \lambda^2\Psi_n^2 + \cdots] = (E_n^O + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots)[E_n^O + \lambda E_n^1 + \lambda^2 E_n^2 + \cdots]$$

$$H^O\Psi_n^O + \lambda\left(H^O\Psi_n^1 + H^{'}\Psi_n^O\right) + \lambda^2\left(H^O\Psi_n^2 + H^{'}\Psi_n^1\right) + \cdots$$
$$= E_n^O\Psi_n^O + \lambda(E_n^O\Psi_n^1 + E_n^1\Psi_n^O) + \lambda^2(E_n^O\Psi_n^2 + E_n^1\Psi_n^1 + E_n^2\Psi_n^O) + \cdots$$

Comparing Coefficient of $\lambda^o, \lambda, \lambda^2$ on both side we have

$$H^O\Psi_n^O = E_n^O\Psi_n^O \tag{3.7}$$

$$H^O\Psi_n^1 + H^{'}\Psi_n^O = E_n^O\Psi_n^1 + E_n^1\Psi_n^O \tag{3.8}$$

$$H^O\Psi_n^2 + H^{'}\Psi_n^1 = E_n^O\Psi_n^2 + E_n^1\Psi_n^1 + E_n^2\Psi_n^O \tag{3.9}$$

3.1.2 First-Order Perturbation Theory

Using above relations we now calculate first-order correction to the energy. Taking inner product the Eq. (3.8) with Ψ_n^O and integrating it we have

$$\langle\Psi_n^O|H^O\Psi_n^1\rangle + \left\langle\Psi_n^O\middle|H^{'}\Psi_n^O\right\rangle = E_n^O\langle\Psi_n^O|\Psi_n^1\rangle + E_n^1\langle\Psi_n^O|\Psi_n^O\rangle$$

As H^O is Hermitian

$$\langle\Psi_n^O|H^O\Psi_n^1\rangle = \langle H^O\Psi_n^O|\Psi_n^1\rangle = E_n^O\langle\Psi_n^O|\Psi_n^1\rangle$$

Therefore

$$E_n^O\langle\Psi_n^O|\Psi_n^1\rangle + \left\langle\Psi_n^O\middle|H^{'}\Psi_n^O\right\rangle = E_n^O\langle\Psi_n^O|\Psi_n^1\rangle + E_n^1\langle\Psi_n^O|\Psi_n^O\rangle$$

$$E_n^1\langle\Psi_n^O|\Psi_n^O\rangle = \left\langle\Psi_n^O\middle|H^{'}\Psi_n^O\right\rangle$$

As $\langle\Psi_n^O|\Psi_n^O\rangle = 1$

$$E_n^1 = \left\langle\Psi_n^O\middle|H^{'}\Psi_n^O\right\rangle \tag{3.10}$$

This is the first order energy correction in the nth state of the system

3.1.3 Second-Order Perturbation Theory

Again taking inner product Eq. (3.9) with Ψ_n^O and integrating we have

$$\langle \Psi_n^O | H^O \Psi_n^2 \rangle + \langle \Psi_n^O | H' \Psi_n^1 \rangle = E_n^O \langle \Psi_n^O | \Psi_n^2 \rangle + E_n^1 \langle \Psi_n^O | \Psi_n^1 \rangle + E_n^2 \langle \Psi_n^O | \Psi_n^O \rangle$$

As H^O is Hermitian

$$\langle \Psi_n^O | H^O \Psi_n^2 \rangle = \langle H^O \Psi_n^O | \Psi_n^2 \rangle = E_n^O \langle \Psi_n^O | \Psi_n^2 \rangle$$

Also $\langle \Psi_n^O | \Psi_n^O \rangle = 1$

Therefore

$$E_n^O \langle \Psi_n^O | \Psi_n^2 \rangle + \langle \Psi_n^O | H' \Psi_n^1 \rangle = E_n^O \langle \Psi_n^O | \Psi_n^2 \rangle + E_n^1 \langle \Psi_n^O | \Psi_n^1 \rangle + E_n^2$$

$$\langle \Psi_n^O | H' \Psi_n^1 \rangle = E_n^1 \langle \Psi_n^O | \Psi_n^1 \rangle + E_n^2$$

Or

$$E_n^2 = \langle \Psi_n^O | H' \Psi_n^1 \rangle - E_n^1 \langle \Psi_n^O | \Psi_n^1 \rangle$$

Now

$$\langle \Psi_n^O | \Psi_n^1 \rangle = \sum_{m \neq n} C_m^{(n)} \langle \Psi_n^O | \Psi_m^O \rangle = 0$$

So

$$E_n^2 = \langle \Psi_n^O | H' \Psi_n^1 \rangle = \sum_{m \neq n} C_m^{(n)} \langle \Psi_n^O | H' \Psi_m^O \rangle = \sum_{m \neq n} \frac{\langle \Psi_m^O | H' \Psi_n^O \rangle \langle \Psi_n^O | H' \Psi_m^O \rangle}{E_n^O - E_m^O}$$

Finally

$$E_n^2 = \sum_{m \neq n} \frac{\left|\langle \Psi_m^O | H' \Psi_n^O \rangle\right|^2}{E_n^O - E_m^O} \tag{3.11}$$

which is the second order correction of energy in the nth state of atom

3.2 Degenerate Perturbation Theory

Suppose we have two orthonormal states Ψ_a^O, Ψ_b^O having same energy in the absence of external perturbation that is

$$H^O \Psi_a^O = E^O \Psi_a^O \tag{3.12}$$

$$H^O \Psi_b^O = E^O \Psi_b^O \tag{3.13}$$

where

$$\langle \Psi_a^O | \Psi_b^O \rangle = 0 \tag{3.14}$$

The linear combination of these two states

$$\Psi^O = \alpha \Psi_a^O + \beta \Psi_b^O \tag{3.15}$$

which is also the eigenstate of H^O having same eigenvalue E^O, therefore

$$H^O \Psi^O = E^O \Psi^O$$

Now let's apply external perturbation H' which splits the unperturbed energy E^O as shown in figure and solve the new Schrödinger Equation

$$H\Psi = E\Psi \tag{3.16}$$

Where

$$H = H^O + \lambda H' \tag{3.17}$$

$$\Psi = \Psi^O + \lambda \Psi^1 + \lambda^2 \Psi^2 + \cdots \tag{3.18}$$

$$E = E^O + \lambda E^1 + \lambda^2 E^2 + \cdots \tag{3.19}$$

Equation (3.16) becomes

$$\left(H^O + \lambda H'\right)[\Psi^O + \lambda \Psi^1 + \lambda^2 \Psi^2 + \cdots] = (E^O + \lambda E^1 + \lambda^2 E^2 + \cdots)[\Psi^O + \lambda \Psi^1 + \lambda^2 \Psi^2 + \cdots]$$

$$H^O\Psi^O + \lambda\left(H^O\Psi^1 + H'\Psi^O\right) + \lambda^2\left(H^O\Psi^2 + H'\Psi^1\right) + \cdots$$
$$= E^O\Psi^O + \lambda(E^O\Psi^1 + E^1\Psi^O) + \lambda^2(E^O\Psi^2 + E^1\Psi^1 + E^2\Psi^O) + \cdots$$

Comparing Coefficient of λ on both sides we obtain

$$H^O\Psi^1 + H'\Psi^O = E^O\Psi^1 + E^1\Psi^O$$

Taking inner product with Ψ_a^O

$$\langle\Psi_a^O|H^O\Psi^1\rangle + \left\langle\Psi_a^O\middle|H'\Psi^O\right\rangle = E^O\langle\Psi_a^O|\Psi^1\rangle + E^1\langle\Psi_a^O|\Psi^O\rangle$$

But H^O is Hermitian so

$$\langle\Psi_a^O|H^O\Psi^1\rangle = \langle H^O\Psi_a^O|\Psi^1\rangle = E^O\langle\Psi_a^O|\Psi^1\rangle$$

Therefore

$$E^O\langle\Psi_a^O|\Psi^1\rangle + \left\langle\Psi_a^O\middle|H'\Psi^O\right\rangle = E^O\langle\Psi_a^O|\Psi^1\rangle + E^1\langle\Psi_a^O|\Psi^O\rangle$$

$$\left\langle\Psi_a^O\middle|H'\Psi^O\right\rangle = E^1\langle\Psi_a^O|\Psi^O\rangle$$

Using equation (3.16) we get

$$\left\langle\Psi_a^O\middle|H'(\alpha\Psi_a^O + \beta\Psi_b^O)\right\rangle = E^1\left\langle\Psi_a^O\middle|\alpha\Psi_a^O + \beta\Psi_b^O\right\rangle$$

$$\alpha\left\langle\Psi_a^O\middle|H'\Psi_a^O\right\rangle + \beta\left\langle\Psi_a^O\middle|H'\Psi_b^O\right\rangle = \alpha E^1\langle\Psi_a^O|\Psi_a^O\rangle + \beta E^1\left\langle\Psi_a^O\middle|\Psi_b^O\right\rangle$$

But

$$\left\langle\Psi_a^O\middle|\Psi_b^O\right\rangle = 0$$

$$\langle\Psi_a^O|\Psi_a^O\rangle = 1$$

Therefore

$$\alpha\left\langle\Psi_a^O\middle|H'\Psi_a^O\right\rangle + \beta\left\langle\Psi_a^O\middle|H'\Psi_b^O\right\rangle = \alpha E^1$$

In more compact form we can write this as

$$\alpha W_{aa} + \beta W_{ab} = \alpha E^1 \tag{3.20}$$

Where

$$W_{aa} = \langle \Psi_a^o | H' \Psi_a^o \rangle$$

$$W_{ab} = \langle \Psi_a^o | H' \Psi_b^o \rangle$$

Similarly inner product with Ψ_a^o gives

$$\alpha W_{ba} + \beta W_{bb} = \beta E^1 \tag{3.21}$$

3.3 DC Stark Effect

When atoms or molecules are placed in external uniform electric field, their spectral lines split up this effect is observed by Johannes Stark in 1913 [11], which is known as Stark effect and the amount of splitting is termed as Stark shift. The amount of splitting is directly proportional to the applied field. This effect had also been observed by Antonino Lo Surdo individually in the same year thus sometimes it is also called Stark-Lo Surdo effect. Stark effect is analogous to Zeeman effect where energy levels split up into various sub-levels in the presence of magnetic field. Stark effect can be explained completely with the help of quantum mechanics. Here we discuss two kind of Stark effects *i.e.* first-order which is linear and the second-order which is quadratic.

3.3.1 Linear Stark Effect

We can illustrate this effect by an example of hydrogen atom, where the Balmer line splits into three components when it is placed in uniform electric field of 10^6 V/cm. We use first order degenerate perturbation theory and quantum numbers to explain linear Stark effect.

For the first excited state of H-atom we have

For n = 2

$$l = 0, m = 0$$

$$l = 1, m = -1,0,1$$

So in the absence of electric field, there are four degenerate states

$$|200>, |210>, |211>, |21\text{-}1>$$

The original Hamiltonian is

$$H_o = \frac{P^2}{2m} - \frac{e^2}{r} \tag{3.22}$$

Now we apply electric field "E" along z-axis, the perturbed Hamiltonian due to the interaction of electron dipole moment "$\vec{d}$" with external electric field $\vec{E} = E\hat{z}$ is

$$H' = -\vec{d}.\vec{E}$$

$$= -e\vec{z}.\vec{E}$$

$$= -ezECos180^o$$

$$= ezE$$

$$= eErCos\theta$$

$$H' = eErCos\theta \tag{3.23}$$

Total Hamiltonian can be written as

$$\hat{H} = H_o + H' \tag{3.24}$$

As E ≥ 10^6 V/cm, so we may neglect spin and all other small effects like Lamb shift, relativistic corrections and hyperfine effects in this Hamiltonian.

To calculate change in energy E′ due to interaction Hamiltonian H′ we solve an eigenvalue equation [12].

$$\hat{H}'|\Psi >= E'|\Psi >$$

$$(\hat{H}' - IE')|\Psi >= 0 \tag{3.25}$$

For non-trivial solution we write above equation in determinant form as

$$\begin{vmatrix} <200|H'|200> - E' & <200|H'|211> & <200|H'|210> & <200|H'|21\text{-}1> \\ <211|H'|200> & <211|H'|211> - E' & <211|H'|210> & <211|H'|21\text{-}1> \\ <210|H'|200> & <210|H'|211> & <210|H'|210> - E' & <210|H'|21\text{-}1> \\ <21\text{-}1|H'|200> & <21\text{-}1|H'|211> & <21\text{-}1|H'|210> & <21\text{-}1|H'|21\text{-}1> - E' \end{vmatrix} = 0$$

$$\langle 200|H'|210\rangle = \langle 210|H'|200\rangle$$

$$= \int R_{21}(r).Y_1^0(\theta,\varphi).\text{eErCos}\theta.R_{20}(r).Y_0^0(\theta,\varphi).r^2\text{Sin}\theta\text{dr}d\theta d\varphi$$

$$= \int \frac{1}{\sqrt{24}} a_o^{-\frac{3}{2}} \frac{r}{a_o} e^{-\frac{r}{2a_o}}.\sqrt{\frac{3}{4\pi}}Cos\theta.\text{eErCos}\theta.\frac{1}{\sqrt{2}}a_o^{-\frac{3}{2}}\left(1-\frac{r}{2a_o}\right)e^{-\frac{r}{2a_o}}.\sqrt{\frac{1}{4\pi}}.r^2\text{Sin}\theta\text{dr}d\theta d\varphi$$

$$= \frac{1}{\sqrt{24}}\sqrt{\frac{3}{4\pi}}\frac{1}{\sqrt{2}}\sqrt{\frac{1}{4\pi}}.\frac{1}{a_o^4}.eE.\int_0^\infty e^{-\frac{r}{a_o}}r^4\left(1-\frac{r}{2a_o}\right)\text{dr}\int_0^\pi Cos^2\theta\text{Sin}\theta\text{d}\theta\int_0^{2\pi}\text{d}\varphi$$

$$= \frac{1}{16\pi}.\frac{1}{a_o^4}.eE.\int_0^\infty e^{-\frac{r}{a_o}}r^4\left(1-\frac{r}{2a_o}\right)\text{dr}.\{-\frac{Cos^3\theta}{3}|_0^\pi.2\pi$$

$$= \frac{1}{16\pi}.\frac{1}{a_o^4}.eE.\int_0^\infty e^{-\frac{r}{a_o}}r^4\left(1-\frac{r}{2a_o}\right)\text{dr}.\frac{2}{3}.2\pi$$

$$= \frac{1}{12a_o^4}.eE.\int_0^\infty e^{-\frac{r}{a_o}}r^4\left(1-\frac{r}{2a_o}\right)\text{dr}$$

$$= \frac{1}{12a_o^4}.eE.\left[\int_0^\infty e^{-\frac{r}{a_o}}r^4\text{dr} - \frac{1}{2a_o}\{\underbrace{r^3\frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - 5\int_0^\infty r^4\frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}\text{dr}\}\right]$$

$$= \frac{1}{12a_o^4}.eE.\left[I - \frac{1}{2a_o}\{5a_o\}I\right] \quad ; \quad I = \int_0^\infty e^{-\frac{r}{a_o}} r^4 \mathrm{dr}$$

$$= \frac{1}{12a_o^4}.eE.\left[-\frac{3}{2}I\right]$$

$$= \frac{-1}{8a_o^4}.eE.[I]$$

$$= \frac{-1}{8a_o^4}.eE.\left[\underbrace{r^4 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - 4\int_0^\infty r^3 \frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}} \mathrm{dr}\right]$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o]\int_0^\infty \left\{e^{-\frac{r}{a_o}}.r^3\right\} \mathrm{dr}$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o]\left[\underbrace{r^3 \frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}|_0^\infty}_{0} - 3\int_0^\infty r^2 \frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}} \mathrm{dr}\right]$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o]\int_0^\infty \left\{e^{-\frac{r}{a_o}}.r^2\right\} \mathrm{dr}$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o]\left[\underbrace{r^2 \frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}|_0^\infty}_{0} - 2\int_0^\infty r \frac{e^{-\frac{r}{a_o}}}{-\frac{r}{a_o}} \mathrm{dr}\right]$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o].[2a_o]\int_0^\infty \left\{e^{-\frac{r}{a_o}}.r\right\} \mathrm{dr}$$

$$= [\frac{-1}{8a_o^4}.eE.[4a_o].[3a_o].[\,2a_o]\left[r\underbrace{\frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}|_0^\infty}_{0} - \int_0^\infty \frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}\mathrm{dr}\right]$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o].[2a_o].[a_o]\int_0^\infty \left\{e^{-\frac{r}{a_o}}\right\}\mathrm{dr}$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o].[2a_o].[a_o].\{\frac{e^{-\frac{r}{a_o}}}{-\frac{1}{a_o}}|_0^\infty$$

$$= \frac{-1}{8a_o^4}.eE.[4a_o].[3a_o].[2a_o].[a_o].[-a_o].\left\{\frac{1}{e^\infty} - \frac{1}{e^0}\right\}$$

$$= \frac{-1}{8a_o^4}.eE.[24a_o^5]$$

$$= -3eEa_O$$

Where remaining all the integrals are zero due to orthogonality condition.

$$\int \Psi^*_{nlm}\Psi_{n'l'm'}\, r^2 Sin\theta dr d\theta d\varphi = \delta_{nn'}\delta_{ll'}\delta_{mm'}$$

Therefore determinant becomes

$$\begin{vmatrix} -E' & 0 & -3eEa_o & 0 \\ 0 & -E' & 0 & 0 \\ -3eEa_o & 0 & -E' & 0 \\ 0 & 0 & 0 & -E' \end{vmatrix} = 0$$

Upon the expansion we have

$$E' = -3eEa_o, 0,0, +3eEa_o \qquad (3.26)$$

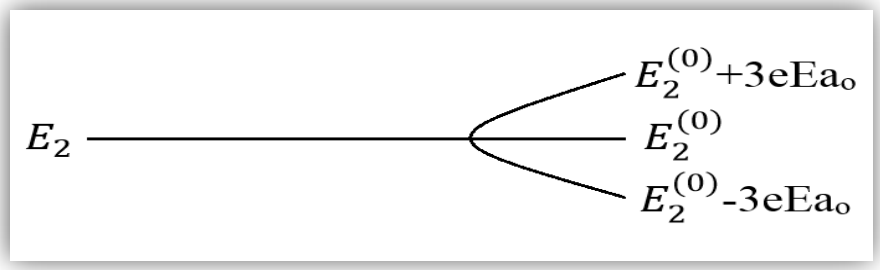

Figure 3.1: Energy splitting for n=2

We may notice here degeneracy has partially lifted with the perturbation used we are left with two degenerate states. The amount of energy splitting has linear dependence on the external electric field.

Thus for $n = 2$ the values of energy of new levels can be written as

$$E_{2^1} = \frac{-13.6}{2^2} - 3\mathrm{eE}a_o \tag{3.27}$$

$$E_{2^2} = \frac{-13.6}{2^2} \tag{3.28}$$

$$E_{2^3} = \frac{-13.6}{2^2} \tag{3.29}$$

$$E_{2^1} = \frac{-13.6}{2^2} + 3\mathrm{eE}a_o \tag{3.30}$$

The wave functions corresponding to these Eigen values are

$$|\Psi_2>_1 = \frac{1}{\sqrt{2}}(|200> + |210>) \tag{3.31}$$

$$|\Psi_2>_2 = \frac{1}{\sqrt{2}}(\ \ |211>) \tag{3.32}$$

$$|\Psi_2>_3 = \frac{1}{\sqrt{2}}(\ \ |21-1>) \tag{3.33}$$

(3.34)

$$|\Psi_2>_4 = \frac{1}{\sqrt{2}}(|200> - |210>)$$

We may write perturbed Hamiltonian in matrix form as

$$H_p = -3eEa_o \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Here we can see that degeneracy has partially lifted, however the states |211>, |21-1> are still degenerate. Spin orbit interactions or other corrections might be invoked to remove degeneracy completely.

3.3.2 Quadratic Stark Effect

For H-atom in ground state, no permanent electric dipole moment exists, so there are only quadratic dependence of energy on external applied electric field which is known as quadratic Stark effect. We see that the ground state of H-atom which is non-degenerate state, no splitting observed after the application of field because for n = 1, we have l = 0, m = 0.

Also first order non-degenerate perturbation theory says

$$E' = eE\langle 100|\hat{z}|100\rangle$$

$$= eE \int |\Psi_{100}(\vec{r})|^2 \, rCos\theta dr^3$$

$= 0$, being an odd integral.

So there is no linear Stark effect for ground state, in such state we observe quadratic Stark effect which is described by second-order non-degenerate perturbation theory.

$$E' = e^2E^2 \sum_{nlm\neq 100} \frac{|\langle nlm|\hat{z}|100\rangle|^2}{E_{100}^{(0)} - E_{nlm}^{(0)}} \tag{3.35}$$

where n = 2, 3, 4 …

Now

$$= \sum_{nlm\neq 100} |\langle nlm|\hat{z}|100\rangle|^2$$

$$= \sum_{nlm} |\langle nlm|\hat{z}|100\rangle|^2$$

$$=< 100|\hat{z}\left\{\sum_{nlm}|nlm><nlm|\right\}\hat{z}|100 >$$

$$= \langle 100|\hat{z}^2|100\rangle$$

$$= \langle 100|r^2Cos^2\theta|100\rangle$$

$$= \int R_{10}(\mathrm{r})Y_0^0(\theta,\varphi).r^2Cos^2\theta.R_{10}(\mathrm{r})Y_0^0(\theta,\varphi).r^2\mathrm{Sin}\theta\mathrm{dr}\mathrm{d}\theta\mathrm{d}\varphi$$

$$= \int |R_{10}(\mathrm{r})|^2.r^2Cos^2\theta.|Y_0^0(\theta,\varphi)|^2.r^2\mathrm{Sin}\theta\mathrm{dr}\mathrm{d}\theta\mathrm{d}\varphi$$

$$= \int \left\{2a_o{}^{-\frac{3}{2}}e^{-\frac{r}{a_o}}\right\}^2.r^2Cos^2\theta.\left\{\frac{1}{4\pi}\right\}.r^2\mathrm{Sin}\theta\mathrm{dr}\mathrm{d}\theta\mathrm{d}\varphi$$

$$= \left\{\frac{1}{4\pi}\right\}\left\{2a_o{}^{-\frac{3}{2}}\right\}^2\int_0^\infty \left\{e^{-\frac{r}{a_o}}\right\}^2 r^4\mathrm{dr}\int_0^\pi Cos^2\theta\mathrm{Sin}\theta\mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\varphi$$

$$= \frac{1}{\pi a_o{}^3}\int_0^\infty \left\{e^{-\frac{2r}{a_o}}r^4\right\}\mathrm{dr}\{-\frac{Cos^3\theta}{3}|_0^\pi.2\pi$$

$$= \frac{4}{3a_o{}^3}\int_0^\infty \left\{e^{-\frac{2r}{a_o}}.r^4\right\}\mathrm{dr}$$

$$= \frac{4}{3{a_o}^3}\left[\underbrace{r^4 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - 4\int_0^\infty r^3 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}\mathrm{dr}\right]$$

$$= \frac{4}{3{a_o}^3}[2a_o]\int_0^\infty \left\{e^{-\frac{2r}{a_o}}.r^3\right\}\mathrm{dr}$$

$$= \frac{4}{3{a_o}^3}[2a_o]\left[\underbrace{r^3 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - 3\int_0^\infty r^2 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}\mathrm{dr}\right]$$

$$= \frac{4}{3{a_o}^3}[2a_o].\left[\frac{3}{2}a_o\right]\int_0^\infty \left\{e^{-\frac{2r}{a_o}}.r^2\right\}\mathrm{dr}$$

$$= \frac{4}{3{a_o}^3}[2a_o].\left[\frac{3}{2}a_o\right]\left[\underbrace{r^2 \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - 2\int_0^\infty r \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}\mathrm{dr}\right]$$

$$= \frac{4}{3{a_o}^3}[2a_o].[\frac{3}{2}a_o].[\,a_o]\int_0^\infty \left\{e^{-\frac{2r}{a_o}}.r\right\}\mathrm{dr}$$

$$= \frac{4}{3{a_o}^3}[2a_o].[\frac{3}{2}a_o].[\,a_o]\left[\underbrace{r \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty}_{0} - \int_0^\infty \frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}\mathrm{dr}\right]$$

$$= \frac{4}{3{a_o}^3}[2a_o].\left[\frac{3}{2}a_o\right].[\,a_o].[\frac{a_o}{2}]\int_0^\infty \left\{e^{-\frac{2r}{a_o}}\right\}\mathrm{dr}$$

$$= \frac{4}{3{a_o}^3}[2a_o].\left[\frac{3}{2}a_o\right].[\,a_o].\left[\frac{a_o}{2}\right]\{\frac{e^{-\frac{2r}{a_o}}}{-\frac{2}{a_o}}|_0^\infty$$

$$= \frac{4}{3{a_o}^3}.\frac{3{a_o}^5}{4}$$

$$= {a_o}^2$$

Now for n = 2

$$E_{100}^{(0)} - E_{nlm}^{(0)} \leq E_{100}^{(0)} - E_{200}^{(0)}$$

As

$$E_{n00}^{(0)} = \frac{-e^2}{8\pi \in_o a_o}\frac{1}{n^2}$$

In atomic units

$$E_{n00}^{(0)} = \frac{-e^2}{2a_o}\frac{1}{n^2}\ where\ k = \frac{1}{4\pi \in_o} = 1$$

Therefore

$$E_{100}^{(0)} - E_{200}^{(0)} = \frac{-e^2}{2a_o} - \frac{-e^2}{8a_o}$$

$$= \frac{-3e^2}{8a_o}$$

Hence

$$E' = e^2E^2\frac{{a_o}^2}{\frac{-3e^2}{8a_o}}$$

$$E' = -\frac{8}{3}a_o^3E^2 \qquad (3.36)$$

which is the formula to calculate the Stark splitting. It is clear from above relation that E' has square dependence on strength of the applied electric field that's why this named as quadratic stark effect.

3.4 AC Stark Effect

Previously we have seen that when an atom is placed in uniform electric field its energy level splits by a significant amount. Now we calculate the shift in energy levels due to time varying electric field. If incident electric field is of angular frequency ω the corresponding change in energy levels is known as AC Stark effect

Consider the external electric field is along Z axis *i.e.*

$$\vec{\mathcal{E}}(t) = \mathcal{E}(t)\hat{z}$$

We also suppose

$$\mathcal{E}(t) = \mathcal{E}_o Cos(\omega t)$$

where $\mathcal{E}_o$ is the amplitude of the electric field. So we may write the interaction Hamiltonian $H'(t)$ as

$$H'(t) = -\mathcal{E}_o D_Z Cos(\omega t) \tag{3.37}$$

In the presence of time varying electric field, $\psi(x,t)$ satisfies the time-dependent Schrödinger Equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial x} = (H_o + H')\psi(x,t) \tag{3.38}$$

Lets expand $\psi(x,t)$ in energy eigenstates in the form of

$$\psi(x,t) = \sum_k C_k \psi_k(x) e^{-iE_k t/\hbar} \tag{3.39}$$

Suppose we switch on electric field at $t = 0$. Before this let atom is in state $a = \gamma^J M_J$. The initial condition is

$$C_k(t) = \delta_{ka} \quad t \leq 0 \tag{3.40}$$

Now we write the amplitude $C_a(t)$ as

$$C_a(t) = |C_a(t)|e^{-i\eta(t)} \tag{3.41}$$

Where η is real phase and $\eta(0) = 0$.

The probability of finding atom in k-state at any time "t" is $|C_a(t)|^2$ which is very small for $k \neq a$ unless the resonance occurs at $\omega = |\omega_{ka}|$ but here we only study the case for which $\omega \neq |\omega_{ka}|$, so that $|C_k(t)| \ll 1$ for $k \neq a$ and $C_a(t) \cong 1$. To understand the significance of phase (t) , we note that the term for which $k = a$ in the expansion

$$C_a(t)\psi_a(x)e^{-\frac{iE_at}{\hbar}} = |C_a(t)|\psi_a(x)e^{-i/\hbar\int_0^t\{E_a+\Delta E_a(t')\}dt'}$$

With

$$\Delta E_a = \hbar\dot{\eta}(t) \tag{3.42}$$

Differentiating Equation (3.16)

$$C_a(t) = \frac{d}{dt}|C_a(t)|e^{-i\eta(t)} - i\dot{\eta}(t)C_a(t)e^{-i\eta(t)} \tag{3.43}$$

Putting this value in above Schrödinger Equation, we get

$$\dot{C}_b(t) = \frac{-i}{\hbar}\sum_k H'_{bk}(t)C_k(t)e^{i\omega_{bk}t'} \tag{3.44}$$

These set of coupled Equations can be solved to second order perturbation $H'(t)$ to get

$$C_a(t) \cong 1 + (i\hbar)^{-2}\sum_{k\neq a}\int_0^t H'_{ak}(t')e^{i\omega_{ak}t'}dt'\int_0^{t'} H'_{ka}(t'')e^{i\omega_{ak}t''}dt'' \tag{3.45}$$

Now

$$\dot{C}_a(t) = (i\hbar)^{-2}\sum_{k\neq a} e^{i\omega_{ak}t'}\int_0^t H'_{ak}(t')e^{i\omega_{ka}t'}dt' \tag{3.46}$$

where $H'_{aa}(t) = 0$

We note that the unperturbed amplitude $C_a^{(0)} = 1$, Therefore

$$\dot{C}_a(t) \approx i\dot{\eta}(t) \tag{3.47}$$

Comparing Equation (3.42) and (3.47)

$$\Delta E_a = -i\hbar \dot{C}_a(t)$$

$$\Delta E_a = (i\hbar)^{-1} \sum_{k \neq a} H'_{ak}(t)\, e^{-i\omega_{ak}t} \int_0^t e^{-i\omega_{ak}t'} H'_{ka}(t')dt'$$

We are interested in the quantity $\Delta\overline{E_a}(t)$, which is the mean value of $\Delta E_a(t)$, *i.e.*

$$\Delta\overline{E_a} = -\frac{E_0^2}{4\hbar} \sum_{k \neq a} |\langle k|D_z|a\rangle|^2 \left\{\frac{1}{\omega_{ka} + \omega} + \frac{1}{\omega_{ka} - \omega}\right\}$$

$$\Delta\overline{E_a} = -\frac{E_0^2}{2\hbar} \sum_{k \neq a} |\langle k|D_z|a\rangle|^2 \left\{\frac{\omega_{ka}}{{\omega_{ka}}^2 - \omega^2}\right\}$$

$$\Delta\overline{E_a} = -\frac{\overline{E^2}}{\hbar} \sum_{k \neq a} |\langle k|D_z|a\rangle|^2 \left\{\frac{\omega_{ka}}{{\omega_{ka}}^2 - \omega^2}\right\}$$

$$\Delta\overline{E_a} = -\frac{1}{2}\overline{E^2}(t)\alpha\left(\gamma^J M_j, \omega\right) \tag{3.48}$$

Where $\alpha(\gamma^J M_j, \omega)$ is dynamic polarizability given by

$$\alpha\left(\gamma^J M_j, \omega\right) = 2 \sum_{\gamma' J'}{}' \frac{\{E_{\gamma' J'} - E_{\gamma J}\} \left|\langle \gamma' J' M_j | D_z | \gamma J M_j \rangle\right|^2}{(E_{\gamma' J'} - E_{\gamma J})^2 - \hbar^2\omega^2} \tag{3.49}$$

Chapter 4

Discrete States of Nucleus

As we see in Atomic Physics when we add one electron in an atom whose valance shell is complete, new shell starts. When one shell closes we see the energy gap there. The ionization energy is very high for closed shell as compared to the incomplete shell. Similar things happen in nucleus, here exist certain **Magic Numbers** that give unique properties to the nucleus. We have following evidences that confirms the existence of shells in nucleus.

- The nuclei having any of the magic number will be extra stable.
- The neutron, proton separation energy for these nuclei is very high for such nuclei.
- The nuclear radius decreases at these numbers.
- Neutron absorption decreases at magic numbers as shown in figure.

These experiments show that the shells exist at magic numbers 2, 8, 20, 28, 50, 82, 126… beyond this we do not have stable nuclei. We search out what is the reason behind that provide extra stability to these nuclei. Different attempts are taken to explain these magic numbers theoretically by solving Schrödinger Equation assuming different nuclear potentials. Some of them are described as follows:

4.1 Shell Model for Simple Harmonic Oscillator Potential

From Schrödinger Equation

$$\left\{\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right\}\Psi = E\Psi \tag{4.1}$$

Where ∇^2 in Spherical Coordinates is

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 Sin\theta}\frac{\partial}{\partial \theta}\left(Sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2}{\partial \varphi^2} \tag{4.2}$$

Also

$$\hat{L}^2 = -\hbar^2\left\{\frac{1}{Sin\theta}\frac{\partial}{\partial \theta}\left(Sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2}{\partial \varphi^2}\right\} \tag{4.3}$$

Therefore

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\frac{\partial}{\partial r}\right\} + \frac{1}{r^2}\left(-\frac{\hat{L}^2}{\hbar^2}\right) \tag{4.4}$$

Hence

$$\left[\frac{-\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\hat{L}^2}{\hbar^2}\right\} + V(r)\right]\Psi = E\Psi$$

Now Let

$$\Psi(r,\theta,\varphi) = R_{nl}(\mathrm{r})Y_l^m(\theta,\varphi) \tag{4.5}$$

Schrödinger Equation (4.1) becomes

$$\left[\frac{-\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\hat{L}^2}{\hbar^2}\right\} + V(r)\right]R_{nl}(\mathrm{r})Y_l^m(\theta,\varphi) = ER_{nl}(\mathrm{r})Y_l^m(\theta,\varphi)$$

$$\frac{-\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\mathrm{R(r)}Y_l^m - \frac{1}{r^2}\frac{\hat{L}^2}{\hbar^2}\mathrm{R(r)}Y_l^m\right\} + V(r)\mathrm{R(r)}Y_l^m = ER_{nl}(\mathrm{r})Y_l^m(\theta,\varphi)$$

$$\frac{-\hbar^2}{2m}\left\{Y_l^m\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\mathrm{R(r)}\right) - \frac{1}{r^2}\frac{\hbar^2 l(l+1)}{\hbar^2}\mathrm{R(r)}Y_l^m\right\} + V(r)\mathrm{R(r)}Y_l^m(\theta,\varphi) = E\mathrm{R(r)}Y_l^m$$

Dividing by $Y_l^m(\theta,\varphi)$ both sides

$$\frac{-\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\mathrm{R(r)}\right) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\mathrm{R(r)} + V(r)\mathrm{R(r)} = E\mathrm{R(r)}$$

$$\frac{-\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\mathrm{R(r)}\right)+\left\{\frac{\hbar^2}{2mr^2}l(l+1)+V(r)\right\}\mathrm{R(r)}=E\mathrm{R(r)} \tag{4.6}$$

Now to find the value of $r^2\frac{\partial}{\partial r}\mathrm{R(r)}$ let's make a substitution.

$$\mathrm{R(r)}=\frac{\mathrm{u(r)}}{r} \tag{4.7}$$

$$\frac{\partial}{\partial r}\mathrm{R(r)}=\frac{\partial}{\partial r}\frac{\mathrm{u(r)}}{r}=-\frac{\mathrm{u(r)}}{r^2}+\frac{1}{r}\frac{\partial u}{\partial r}$$

$$\Rightarrow r^2\frac{\partial}{\partial r}\mathrm{R(r)}=r^2\left\{-\frac{\mathrm{u(r)}}{r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right\}$$

$$=-\mathrm{u(r)}+r\frac{\partial u}{\partial r}$$

$$\Rightarrow\frac{\partial}{\partial r}\left\{r^2\frac{\partial\mathrm{R(r)}}{\partial r}\right\}=\frac{\partial}{\partial r}\left\{-\mathrm{u(r)}+r\frac{\partial u}{\partial r}\right\}$$

$$\Rightarrow\frac{\partial}{\partial r}\left\{r^2\frac{\partial\mathrm{R(r)}}{\partial r}\right\}=-\frac{du}{dr}+r\frac{d^2u}{dr^2}+\frac{du}{dr}(1)$$

$$\Rightarrow\frac{\partial}{\partial r}\left\{r^2\frac{\partial\mathrm{R(r)}}{\partial r}\right\}=r\frac{d^2u}{dr^2} \tag{4.8}$$

Put this value in Eq.(4.6) we have

$$\frac{-\hbar^2}{2mr^2}\left(r\frac{d^2u}{dr^2}\right)+\left\{\frac{\hbar^2}{2mr^2}l(l+1)+V(r)\right\}\mathrm{R(r)}=E\mathrm{R(r)}$$

$$\frac{-\hbar^2}{2mr}\frac{d^2u}{dr^2}+\left\{\frac{\hbar^2}{2mr^2}l(l+1)+V(r)\right\}\frac{u(\mathrm{r})}{r}=E\frac{u(\mathrm{r})}{r}$$

$$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2}+\left\{\frac{\hbar^2}{2mr^2}l(l+1)+V(r)\right\}\mathrm{u(r)}=E\mathrm{u(r)}$$

$$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2}+\frac{\hbar^2}{2mr^2}l(l+1)u(\mathrm{r})+V(r)u(\mathrm{r})=Eu(\mathrm{r}) \tag{4.9}$$

For harmonic oscillator we take

$$V(r)=\frac{1}{2}kr^2=\frac{1}{2}m\omega^2r^2 \tag{4.10}$$

Therefore

$$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2}+\frac{\hbar^2}{2mr^2}l(l+1)u(\mathrm{r})+\frac{1}{2}m\omega^2r^2u(\mathrm{r})=Eu(\mathrm{r}) \tag{4.11}$$

Now Put

$$x=\frac{r}{a}\quad or\quad r=xa\quad where\quad a=\sqrt{\frac{\hbar}{m\omega}} \tag{4.12}$$

$$\Rightarrow\frac{1}{r^2}=\frac{1}{x^2a^2}=\frac{m\omega}{\hbar x^2} \tag{4.13}$$

$$\Rightarrow r^2=x^2a^2=\frac{\hbar x^2}{m\omega} \tag{4.14}$$

$$\Rightarrow\frac{d^2u}{dr^2}=\frac{d^2u}{d(x^2a^2)}=\frac{d^2u}{dx^2}\left(\frac{1}{a^2}\right)=\left(\frac{1}{a^2}\right)\frac{d^2u}{dx^2}=\frac{m\omega}{\hbar}\frac{d^2u}{dx^2} \tag{4.15}$$

Using (4.13),(4.14),(4.15) in Eq.(4.11) We have

$$\frac{-\hbar^2}{2m}\frac{m\omega}{\hbar}\frac{d^2u(x)}{dx^2}+\frac{\hbar^2}{2m}\frac{m\omega}{\hbar x^2}l(l+1)u(\mathrm{x})+\frac{1}{2}m\omega^2\frac{\hbar x^2}{m\omega}u(x)=Eu(\mathrm{x})$$

$$\left(\frac{-\hbar\omega}{2}\right)\frac{d^2u}{dx^2}+\frac{l(l+1)u}{x^2}\left(\frac{\hbar\omega}{2}\right)+x^2u\left(\frac{\hbar\omega}{2}\right)=Eu$$

Or

$$\frac{d^2u}{dx^2}-\frac{l(l+1)u}{x^2}-x^2u=\frac{-Eu}{\frac{\hbar\omega}{2}} \tag{4.16}$$

Put

$$\frac{E}{\hbar\omega/2} = \epsilon \tag{4.17}$$

$$\frac{d^2u}{dx^2} - \frac{l(l+1)u}{x^2} - x^2 u = -\epsilon\, u$$

$$\frac{d^2u}{dx^2} - \frac{l(l+1)u}{x^2} - x^2 u + \epsilon\, u = 0 \tag{4.18}$$

We try a solution

$$u(x) = x^{l+1}e^{-\alpha x^2} \tag{4.19}$$

$$\Rightarrow \frac{d^2u}{dx^2} = x^{l+3}(4\alpha^2)e^{-\alpha x^2} + x^{l+1}\{(-2\alpha)(2l+3)\}e^{-\alpha x^2} + x^{l-1}l(l+1)e^{-\alpha x^2} \tag{4.20}$$

Using $u(x), \frac{d^2u}{dx^2}$ in Eq. (4.18)

$$x^{l+3}(4\alpha^2)e^{-\alpha x^2} + x^{l+1}\{(-2\alpha)(2l+3)\}e^{-\alpha x^2} + x^{l-1}l(l+1)e^{-\alpha x^2} - \frac{l(l+1)}{x^2}x^{l+1}e^{-\alpha x^2}$$
$$- x^2 x^{l+1}e^{-\alpha x^2} + \epsilon\, x^{l+1}e^{-\alpha x^2} = 0$$

$$[x^{l+3}(4\alpha^2 - 1) + x^{l+1}\{(-2\alpha)(2l+3) + \epsilon\} + x^{l+1}\{l(l+1) - l(l+1)\}]e^{-\alpha x^2} = 0$$

$$As \quad e^{-\alpha x^2} \neq 0$$

$$x^{l+3}(4\alpha^2 - 1) + x^{l+1}\{(-2\alpha)(2l+3) + \epsilon\} = 0 \tag{4.21}$$

Comparing powers of "x" on both sides. We have

$$(4\alpha^2 - 1) = 0 \quad \Rightarrow \alpha = \frac{1}{2} \tag{4.22}$$

&

$$(-2\alpha)(2l+3) + \epsilon = 0$$

$$-2(1/2)(2l+3) + \epsilon = 0$$

$$-(2l+3) + \epsilon = 0$$

$$\in = 2l + 3$$

Or

$$\frac{E}{\hbar\omega/2} = 2l + 3$$

$$E = \frac{2l+3}{2}\hbar\omega \; ; l = 0,1,2,3 \ldots \tag{4.23}$$

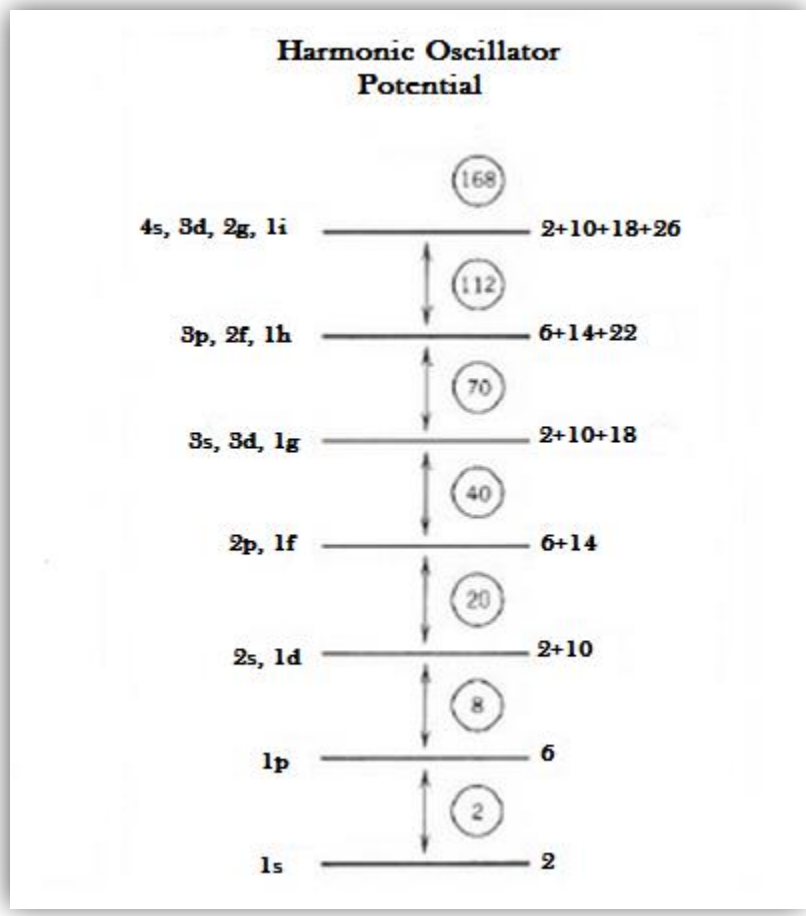

Figure 4.1: Nuclear energy levels for harmonic oscillator potential

So simple harmonic oscillator potential was not our good assumption as it fails to explain the magic numbers except first three. Let's try with Infinite Square well potential.

4.2 Shell Model for Infinite Square Well Potential

We assume that if we pick up one neutron/proton from nucleus, it experiences square well potential created by the rest of nucleons. This potential is described as

$$V = O \quad for \quad r < r_o$$

$$V = \infty \quad for \quad r > r_o$$

This is central potential because it does not depend upon θ, φ. Now again from Schrödinger Equation in the form of

$$\left[\frac{-\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{r^2}\frac{\hat{L}^2}{\hbar^2}\right\} + V(r)\right]\Psi = E\Psi \tag{4.24}$$

$$\Psi(r, \theta, \varphi) = \frac{\mathrm{u(r)}}{r} Y_l^m(\theta, \varphi) \tag{4.25}$$

For $r < r_o$ we have

$$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} + \left\{\frac{\hbar^2 l(l+1)}{2mr^2}\right\}\mathrm{u(r)} = E\mathrm{u(r)} \tag{4.26}$$

Solution of the above equation is Bessel function

$$u = \gamma j_l(kr) \tag{4.27}$$

Where

$$j_l(kr_o) = 0 \; ; k = \sqrt{\frac{2mE}{\hbar^2}} \tag{4.28}$$

Here m is the mass of neutron/proton

$$for\ l = 0 \quad s - state \quad n = 1, 2, 3 \ldots$$

$$for\ l = 1 \quad p - state \quad n = 1, 2, 3 \ldots$$

$$for\ l = 2 \quad d - state \quad n = 1, 2, 3 \ldots$$

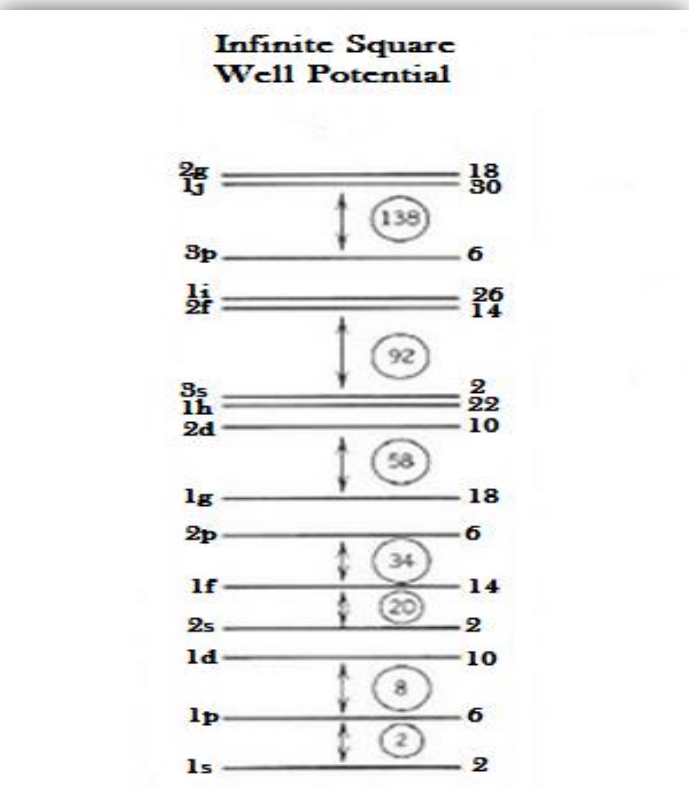

Figure 4.2: Nuclear energy levels for Infinite Square well potential

We are making a mistake, we are trying to explain magic numbers with infinite potentials which are both unrealistic as we see neutron/proton separation energies are finite also nuclear potential is not like sharp edges as we have taken in infinite square well potential that's why both potentials could not explain the magic numbers completely.

4.3 Shell Model for Woods-Saxon Potential

We make an attempt with this nuclear potential which is more realistic as compared to the infinite square well potential and harmonic oscillator potential because this is finite potential having round edges. Mathematically it can be expressed as

$$V = \frac{-V_o}{1 + e^{(r-R)/a}} \tag{4.29}$$

where $V_o = 50MeV$, $a = .42fm$, $R = R_O A^{1/3}$

This potential was also tried to explain the magic numbers but all in vain. Again Scientists brought the idea of spin-orbit coupling from atomic physics to explain magic numbers theoretically.

$$\vec{J} = \vec{L} + \vec{S} \tag{4.30}$$

$$J^2 = L^2 + S^2 + 2\vec{L}.\vec{S}$$

Or

$$\vec{L}.\vec{S} = \frac{J^2 - L^2 - S^2}{2}$$

Taking average

$$\langle \vec{L}.\vec{S} \rangle = \frac{\langle J^2 - L^2 - S^2 \rangle}{2} \tag{4.31}$$

If $\vec{L} = 0$, $\vec{J} = 0 + \frac{1}{2} = \frac{1}{2}$

If $\vec{L} \neq 0$, $\vec{J} = \vec{L} + \frac{1}{2}$, $\vec{L} - \frac{1}{2}$

For $\vec{J} = \vec{L} + \frac{1}{2}$

$$\langle \vec{L}.\vec{S} \rangle = \frac{\langle J^2 - L^2 - S^2 \rangle}{2}$$

$$= \frac{\left(l + \frac{1}{2}\right)\left(l + \frac{3}{2}\right)\hbar^2 - l(l+1)\hbar^2 - \frac{1}{2}(\frac{1}{2} + 1)\hbar^2}{2}$$

$$= \frac{\hbar^2}{2}\left\{l^2 + 2l + \frac{3}{4} - l^2 - l - \frac{3}{4}\right\}$$

$$\langle \vec{L}.\vec{S} \rangle = \frac{l\hbar^2}{2} \tag{4.32}$$

For $\vec{J} = \vec{L} - \frac{1}{2}$

$$\langle \vec{L}.\vec{S} \rangle = \frac{\langle J^2 - L^2 - S^2 \rangle}{2}$$

$$= \frac{\left(l + \frac{1}{2}\right)\left(l - \frac{1}{2}\right)\hbar^2 - l(l+1)\hbar^2 - \frac{1}{2}\left(\frac{1}{2} + 1\right)\hbar^2}{2}$$

$$= \frac{\hbar^2}{2}\left\{l^2 - \frac{1}{4} - l^2 - l - \frac{3}{4}\right\}$$

$$\langle \vec{L}.\vec{S} \rangle = \frac{-\hbar^2}{2}\{l + 1\} \tag{4.33}$$

Splitting $\frac{\hbar^2}{2}\{2l + 1\}$

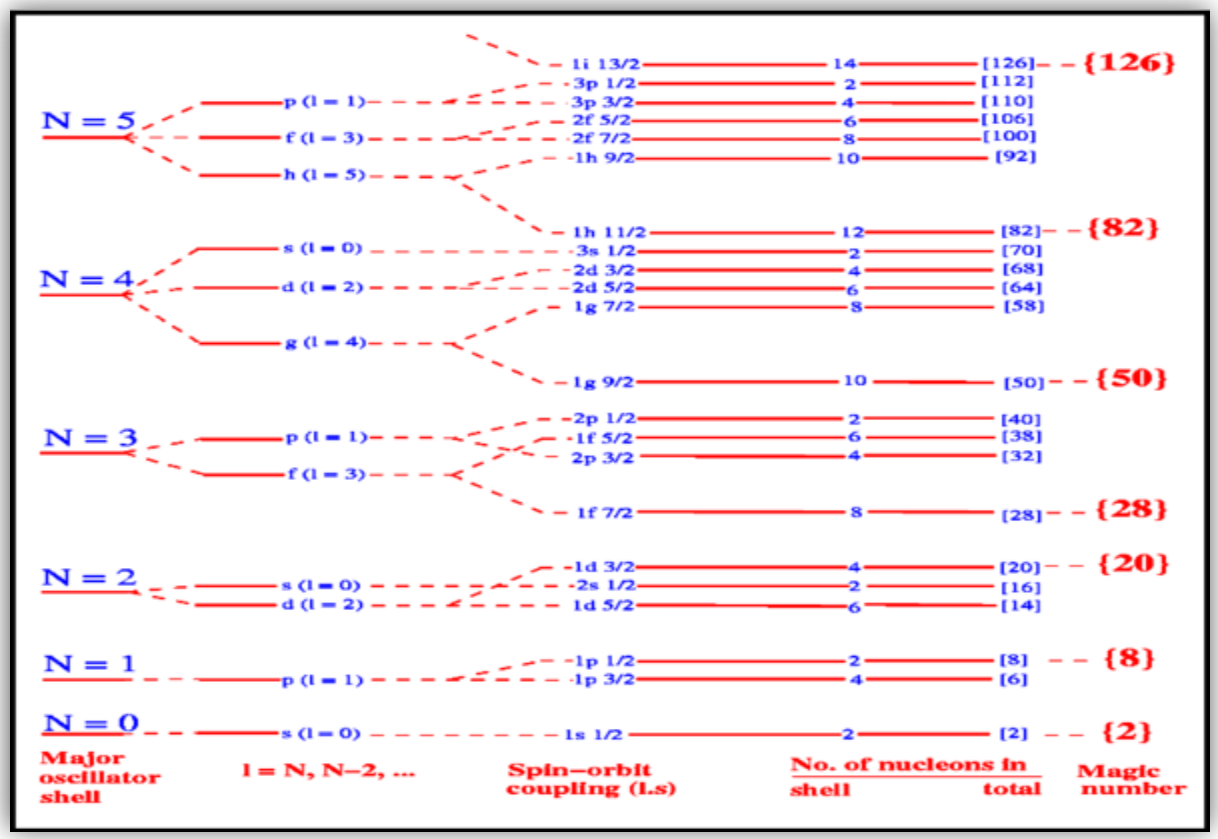

Figure 4.3: Woods-Saxon Potential

Chapter 5

Nuclear Stark Effect

In the previous chapter we have learnt in detail that nucleus also possesses the definite energy level being a bound state. We want to study the changes in these energy levels as it is subjected to a laser. As the wavelength of laser is not comparable to the size of the nucleus, therefore it is not possible for us to probe the nucleus with the help of modern laser facilities. But we can interact with the nucleus indirectly that if we shine laser upon a Hydroden-like system, due to the oscillation of the electron in laser field nucleus experiences periodic electric field due to its oscillation. We will study how this field changes the energy levels of the nucleus.

As we have established the formulism for AC Stark effect in chapter 3 at atomic level.

$$\Delta\overline{E_a} = -\frac{1}{2}\overline{\mathcal{E}^2}(t)\alpha\left(\gamma^J M_j, \omega\right) \tag{5.1}$$

$$\Delta\overline{E_a} = -\frac{1}{2}\overline{\mathcal{E}^2}(t)\left[2\sum_{\gamma'J'}{}' \frac{\{E_{\gamma'J'} - E_{\gamma J}\}\left|\langle\gamma'J'M_j|D_z|\gamma JM_j\rangle\right|^2}{(E_{\gamma'J'} - E_{\gamma J})^2 - \hbar^2\omega^2}\right]$$

$$\Delta\overline{E_a} = -\overline{\mathcal{E}^2}(t)\sum_{\gamma'J'}{}' \frac{\{E_{\gamma'J'} - E_{\gamma J}\}\left|\langle\gamma'J'M_j|D_z|\gamma JM_j\rangle\right|^2}{\left(E_{\gamma'J'} - E_{\gamma J}\right)^2 - \hbar^2\omega^2} \tag{5.2}$$

Where $\alpha(\gamma^J M_j, \omega)$ is dynamic polarizability given by

$$\alpha\left(\gamma^J M_j, \omega\right) = 2\sum_{\gamma'J'}{}' \frac{\{E_{\gamma'J'} - E_{\gamma J}\}\left|\langle\gamma'J'M_j|D_z|\gamma JM_j\rangle\right|^2}{\left(E_{\gamma'J'} - E_{\gamma J}\right)^2 - \hbar^2\omega^2} \tag{5.3}$$

5.1 Electric field experienced by nucleus under the action of laser beam

The electric field of laser can be supposed in the form in the dipole approximation

$$\mathcal{E}(t) = \mathcal{E}_o e^{-i\omega t} \tag{5.4}$$

Equation of motion of electron in the presence of electric field is

$$m\frac{dv}{dt} = -e\mathcal{E}(t)$$

$$m\frac{d}{dt}v_o e^{-i\omega t} = -e\mathcal{E}(t)$$

$$mv(-i\omega) = -e\mathcal{E}(t)$$

$$v(t,\omega) = \frac{e\mathcal{E}(t)}{im\omega}$$

$$\frac{d \propto}{dt} = \frac{e\mathcal{E}(t)}{im\omega}$$

$$\frac{d}{dt} \propto_o e^{-i\omega t)} = \frac{e\mathcal{E}(t)}{im\omega}$$

$$\propto (-i\omega) = \frac{e\mathcal{E}(t)}{im\omega}$$

$$\propto (t,\omega) = \frac{e\mathcal{E}(t)}{m\omega^2} \tag{5.5}$$

This is the oscillating amplitude of electron. It can also be written as [13]

$$\propto (t,\omega) = \frac{e\mathcal{E}_o Sin\omega t}{m(\omega_o^2 - \omega^2)} \tag{5.6}$$

where ω is the frequency of laser and ω_o is the frequency of oscillating electron.

In laser field, the force between electron and proton can be written as the function of “t” and "ω"where $\omega$is the frequency of laser.

The field experienced by nucleus is

$$\mathcal{E}(t,\omega) = \frac{ke}{r^2(t,\omega)} \tag{5.7}$$

When electron at maximum distance from nucleus

$$\mathcal{E}_1(t,\omega) = \frac{ke}{\{r_o + \propto (t,\omega)\}^2} \tag{5.8}$$

When electron at minimum distance from nucleus

$$\mathcal{E}_2(t,\omega) = \frac{ke}{\{r_o - \propto (t,\omega)\}^2} \tag{5.9}$$

where r_o is Bohr radius.

The average field is

$$\mathcal{E}_{av}(t,\omega) = \frac{\mathcal{E}_1 + \mathcal{E}_2}{2}$$

$$\mathcal{E}_{av}(t,\omega) = -\frac{1}{2}\left[\frac{ke}{\{r_o - \propto (t,\omega)\}^2} + \frac{ke}{\{r_o + \propto (t,\omega)\}^2}\right] \tag{5.10}$$

This is average electric field experienced by nucleus due to the electron oscillation under the action of laser beam. Put this value in Eq(5.2)

$$\Delta\overline{E_a} = -\left\{-\frac{1}{2}\left[\frac{ke}{\{r_o - \propto (t,\omega)\}^2} + \frac{ke}{\{r_o + \propto (t,\omega)\}^2}\right]\right\}^2 \sum_{\gamma' J'}{}' \frac{\{E_{\gamma' J'} - E_{\gamma J}\}\left|\langle \gamma' J' M_j | D_z | \gamma J M_j \rangle\right|^2}{(E_{\gamma' J'} - E_{\gamma J})^2 - \hbar^2\omega^2}$$

$$\Delta\overline{E_a} = -\frac{1}{4}\left[\frac{ke}{\{r_o - \propto (t,\omega)\}^2} + \frac{ke}{\{r_o + \propto (t,\omega)\}^2}\right]^2 \sum_{\gamma' J'}{}' \frac{\{E_{\gamma' J'} - E_{\gamma J}\}\left|\langle \gamma' J' M_j | D_z | \gamma J M_j \rangle\right|^2}{(E_{\gamma' J'} - E_{\gamma J})^2 - \hbar^2\omega^2}$$

Using $\propto (t,\omega) = \frac{e\mathcal{E}(t)}{m(\omega_o^2 - \omega^2)}$

$$\Delta\overline{E_a} = -\frac{1}{4}\left[\frac{ke}{\left\{r_o - \frac{e\mathcal{E}_o Sin\omega t}{m(\omega_o^2 - \omega^2)}\right\}^2} + \frac{ke}{\left\{r_o + \frac{e\mathcal{E}_o Sin\omega t}{m(\omega_o^2 - \omega^2)}\right\}^2}\right]^2 \sum_{\gamma' J'}{}' \frac{\{E_{\gamma' J'} - E_{\gamma J}\}\left|\langle \gamma' J' M_j | D_z | \gamma J M_j \rangle\right|^2}{(E_{\gamma' J'} - E_{\gamma J})^2 - \hbar^2\omega^2}$$

Now the expectation value of electric dipole moment of H-atom is

$$\langle \gamma' J' M_j | D_z | \gamma J M_j \rangle = e r_o$$

Therefore

$$\Delta\overline{E_a} = -\frac{1}{4}\left[\frac{ke}{\left\{r_o - \frac{e\varepsilon_o Sin\omega t}{m(\omega_o^2-\omega^2)}\right\}^2} + \frac{ke}{\left\{r_o + \frac{e\varepsilon_o Sin\omega t}{m(\omega_o^2-\omega^2)}\right\}^2}\right]^2 \sum_{\gamma' J'}{}' \frac{\{E_{\gamma' J'} - E_{\gamma J}\}(e r_o)^2}{(E_{\gamma' J'} - E_{\gamma J})^2 - \hbar^2\omega^2}$$

In atomic units

$$k = m = \hbar = e = r_o = 1$$

$$\Delta\overline{E_a} = -\frac{1}{4}\left[\frac{1}{\left\{1 - \frac{\varepsilon_o Sin\omega t}{(\omega_o^2-\omega^2)}\right\}^2} + \frac{1}{\left\{r_o + \frac{\varepsilon_o Sin\omega t}{(\omega_o^2-\omega^2)}\right\}^2}\right]^2 \sum_{\gamma' J'}{}' \frac{E_{\gamma' J'} - E_{\gamma J}}{\left(E_{\gamma' J'} - E_{\gamma J}\right)^2 - \omega^2} \tag{5.11}$$

5.2 Restrictions on laser parameters

We can vary the laser parameters ε_o and ω such that oscillating electron neither collide with the nucleus nor ionize. Necessary condition to avoid electron-nucleus collision is $\propto << r_o$where $r_o$the Bohr radius and$\propto$ is the amplitude of oscillation of the electron. So we can adjust ε_o and ω such that oscillating amplitude do not exceed the Bohr radius that is

$$\propto << r_o$$

Or

$$\frac{\varepsilon_o Sin\omega t}{(\omega_o^2 - \omega^2)} < 1 \tag{5.12}$$

The ionization energy of H-atom is 0.499a.u. So we bound our self to take ω less than 0.499a.u. to prevent system to ionize that is

$$\omega < 0.499 a.u \tag{5.13}$$

The system will not be ionized.

There is another important thing to keep in mind that our dealing is purely non-relativistic, so if the relativistic factor

$$q = \frac{\varepsilon_o}{emc} < 1$$

Or

$$q = \frac{\varepsilon_o}{137} < 1 \tag{5.14}$$

We may ignore all relativistic effects in the motion of electron.

Now let's see the Stark splitting for two nuclear states of H-atom[14]

$$E_{\gamma'J'} - E_{\gamma J} = 800keV$$

$$= \frac{800x10^3}{27.214} a.u$$

$$E_{\gamma'J'} - E_{\gamma J} = 2.94 * 10^4 a.u \tag{5.15}$$

Eq.(5.11) reduces to

$$\Delta\overline{E_a} = -\frac{1}{4}\left[\frac{1}{\left\{1 - \frac{\varepsilon_o Sin\omega t}{(\omega_o^2 - \omega^2)}\right\}^2} + \frac{1}{\left\{1 + \frac{\varepsilon_o Sin\omega t}{(\omega_o^2 - \omega^2)}\right\}^2}\right]^2 \left\{\frac{2.94 * 10^4}{(2.94x10^4)^2 - \omega^2}\right\} \tag{5.15}$$

5.3 Discussion on results

Graph 5.1 shows the variation of Stark shift when we change the frequency of incident laser

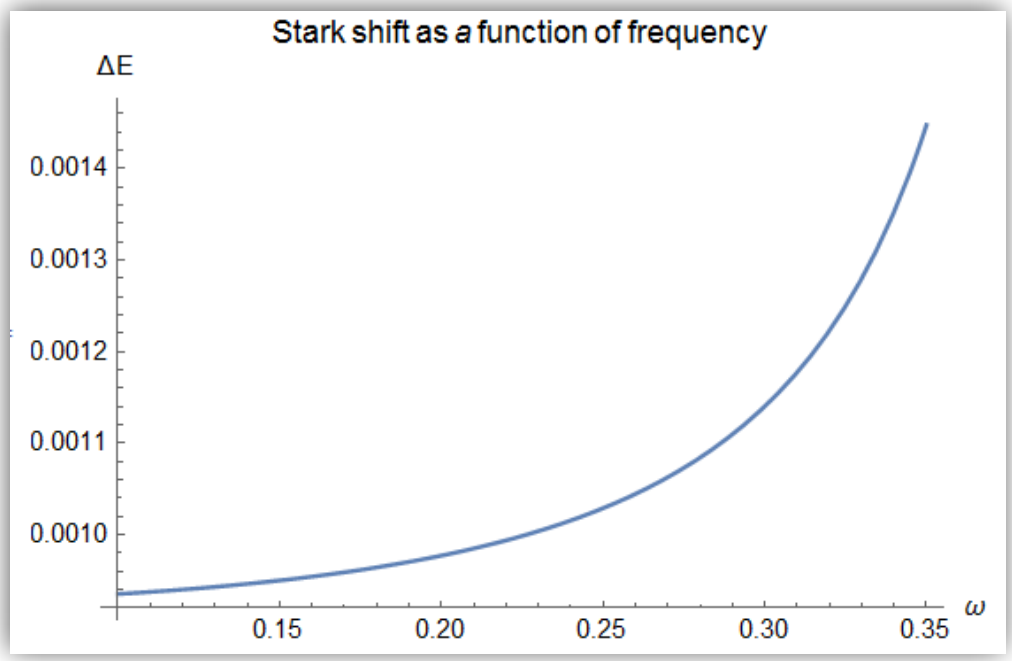

Figure 5.1: Variation of stark shift with frequency

It is clear from the graph that Stark shift increases as we increase laser frequency because when we increase frequency, the number of interactions per second of electron with nucleus increases hence the electric field per second experienced by the nucleus increases due to which shift in the energy levels in H-atom nucleus increases. In other words we can say that electron remains in contact with the nucleus for greater time therefore there is increase in Stark shift at higher frequency.

When we fix the frequency and vary the strength of electric field of external laser, electron will come more close to the nucleus and it experiences greater magnitude of electric field hence Stark shift again increases as shown in the figure 5.2

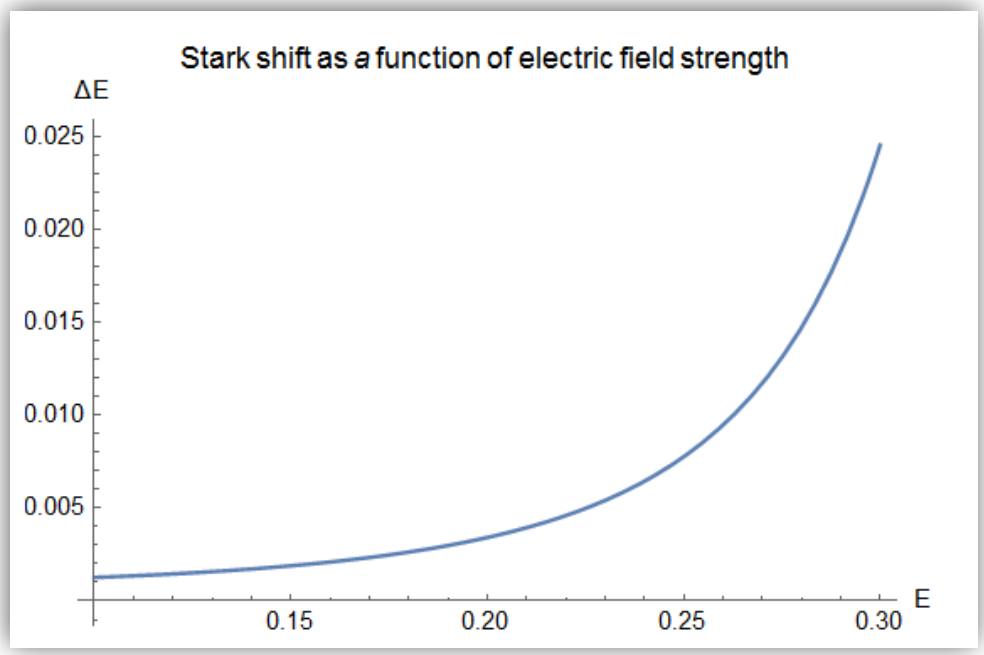

Figure 5.2: Variation of stark shift with electric field strength

If we keep frequency and field strength of the laser unchanged and vary the laser time, figure 5.3 shows that Stark shift is oscillating about a fixed value as we change laser time from 1fs to 5fs. By changing of laser time we mean that we have not changed the laser but we have increased the number of active cycles of the laserand rest of the cycles are inactive or silent.

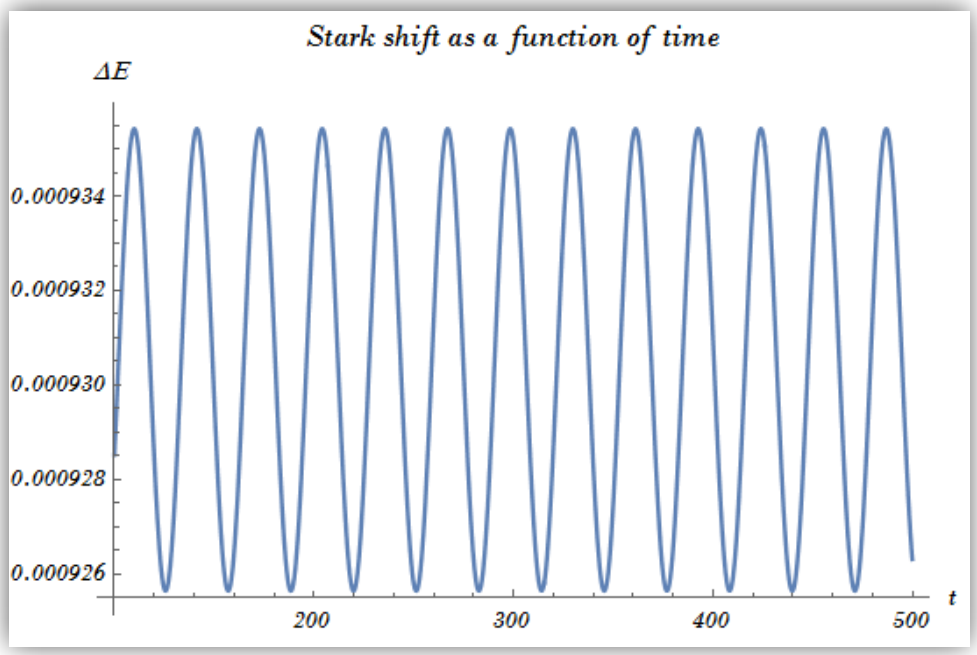

Figure 5.3: Variation of stark shift with time

The relation $E(t) = E_O Sin\omega t$ shows that with the increase of time, the electric field $E(t)$ will oscillate hence there are sinusoidal oscillations of Stark shift in this case.

Referances ;

[1] Ch. Aldarmaa1, L. Khenmedekh and O. Lkhagva, International Journal of Mathematical Archive-3(7), (2012)

[2] H. G. Muller, H. B. van Linden van den Heuvell, P. Agostini, G. Petite, A. Antonetti, M. Franco, and A. Migus,Phys. Rev. Lett. 60, 565(1987).

[3] R. R. Freeman, P. H. Bucksbaum, H. Milchberg, S. Darack, D. Schumacher, and M. E. Geusic, Phys. Rev. Lett. 59, 1092 (1987).

[4] L. V. keldysh, Sov. Phys. J.E.T.P, 20, 1307 (1965).

[5] R. W. Boyd, Non-linear Optics, Academic Press, London, UK (2008).

[6] S. Augst, D. Strickland, D. D. Meyerhofer, S. L. Chin, and J. H. Eberly, Phys. Rev. Lett. 63, 2212 (1989).

[7] P. A. Franken, A. E. Hill, C.W. Peters and G. Weinreich, Phys. Rev. Lett. 7, 118 (1961)

[8] H. Hu, Multi-photon creation and single-photon annihilation of electron-positron pairs, Thesis (PhD), University of Heidelberg, Germany, (2011).

[9] N. H. Burnett et al., Appl. Phys. Lett., 31, 172 (1977).

[10] P. M. Paul, E. S. Toma, P. Breger, F. Augé Ph. Balcou, H. G. Muller, P. Agostini, Science(2001)

[11] A. J. Kox, Ann. Phys. (Berlin) 525, No. 5, A63–A66 (2013).

[12] D. J. Griffith, "Introduction to Quantum Mechanics", Pearson Prentice Hall(2005).

[13] Y. I. Salamin et al. phys. Rep. 417, 41(2006).

[14] J. E. Evans, C. W. Malich, and J. R. Risser, Phys. Rev. 75, 1161(1949).